Option Valuation

Arbitrage Limit Theorems indicate premium boundaries and relative value for call and put options – A range of values, but not an exact pricing.

Option Characteristics that make Valuation Difficult:

Options are time dependent

Future Cash flows are unknown – dependent on the future value of the underlying stock, possibly payment of dividends

Options are different from other financial assets in that they offer a fixed exercise price, limited loss to the purchaser, and a variety of choices (time, strike price, right to buy or sell stock)

Binomial Option Pricing Model

Analyzes Option pricing by viewing the Stock price as having one of two possible outcomes.

One Period Binomial Option Pricing

Only one period until expiration

2 outcomes ① Stock goes up by a given %
② Stock goes down by a certain %

Assume Home Depot's Stock currently trades at \$42/Shr. One period from now we consider that the ① price goes up 6%, $S_u = 42(1.06) = \$44.52$
or ② price goes down 7%, $S_d = 42(1-.07) = \$39.06$

Consider a call that has a strike price of \$40 and expires in one period

$$C_u = \text{Max}\,[0, S_u - X] = \$44.52 - \$40 = \$4.52$$

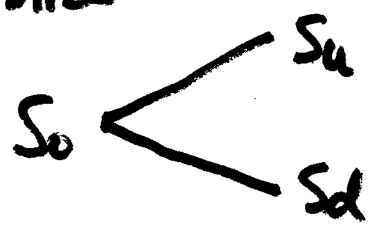
or

$$C_d = \text{Max.}\,[0, S_d - X] = 0$$

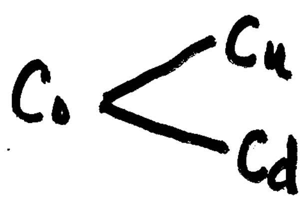
out of the money

$$\$39.06 - \$40. < 0$$

Underlying Stock Price



Call Price



Need to determine the probabilities of occurrence for the decision tree path.

Weights Using a Uniform Distribution

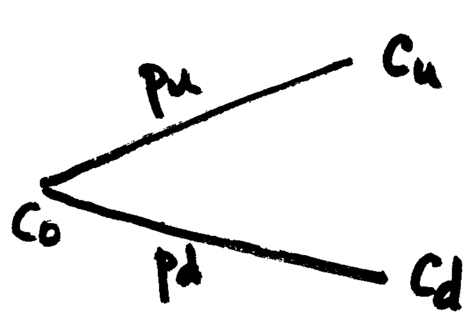
- Risk-neutral prob. of stock price
52 increasing, Pu

$$P_u = \frac{r_f - d}{u - d}$$

r_f = risk free rate, d = % decrease in stock price
u = % increase in stock price

- $P_u + P_d = 1$ where P_d is the risk-neutral prob. of stock price decreasing

$$P_d = 1 - P_u = 1 - \frac{r_f - d}{u - d} = \frac{u - r_f}{u - d}$$



Expected Value of the Call, C_0

$$C_0 = \frac{1}{1+r_f}\left\{ P_u C_u + P_d C_d \right\}$$

If the risk free rate, $r_f = 4\%$

$$P_u = \frac{.04-(-.07)}{.06-(-.07)}$$

$P_u = 84.62\%$

$P_d = 1 - 84.62\% = 15.38\%$

$$C_0 = \frac{1}{1.04}\left[84.62\%(\$4.52) + 15.38\%(0) \right]$$

$C_0 = \$3.68$

⇒ If the call is priced above or below \$3.68 an arbitrage possibility would exist.

One Period Hedged Portfolio

Let H_0 = # shares of stock held in a portfolio that needs to be hedged

Idea is to have $H_0 S_0$, the stock portfolio [long position] hedged by a [short] call position, C_0.

The value of the portfolio: $V_0 = H_0 S_0 - C_0$

- Two possible outcomes: Either the stock rises $S_0 \to S_u$
  or the stock falls $S_0 \to S_d$

Two possible valuations for the portfolio:

$$V_u = H_0 S_u - C_u$$
$$V_d = H_0 S_d - C_d$$

You want to determine H_0 (simple hedge ratio) such that $V_u = V_d$ i.e. the value of the portfolio remains the same regardless of whether the stock goes up or down.

If $V_u = V_d$ then $H_0 S_u - C_u = H_0 S_d - C_d$

$$\Rightarrow H_0 (S_u - S_d) = C_u - C_d$$

$$H_0 = \frac{C_u - C_d}{S_u - S_d}$$

E.G. Using our Home Depot data:

$$H_0 = \frac{(4.52 - 0)}{(44.52 - 39.06)} = .8278$$

$\Rightarrow$ Each shorted call will cover .8278 shares of HD stock.

$V_u = .8278(44.52) - 4.52 = \$32.33$

Stock rises 6% from \$42 $\rightarrow$ \$44.52

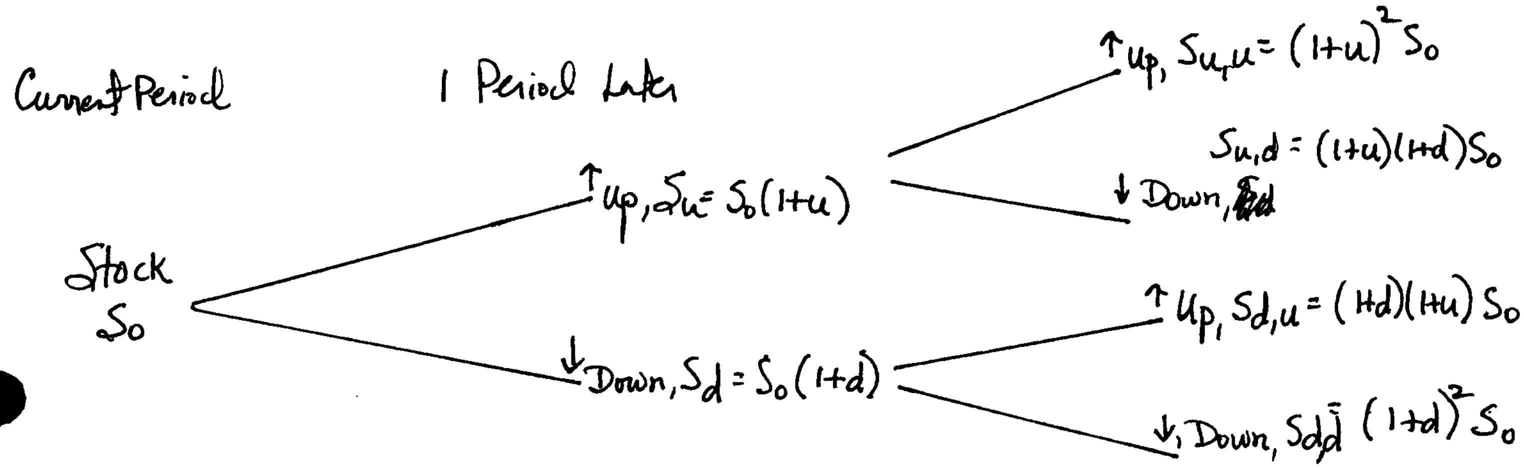
$V_d = .8278(39.06) - 0 = \$32.33$

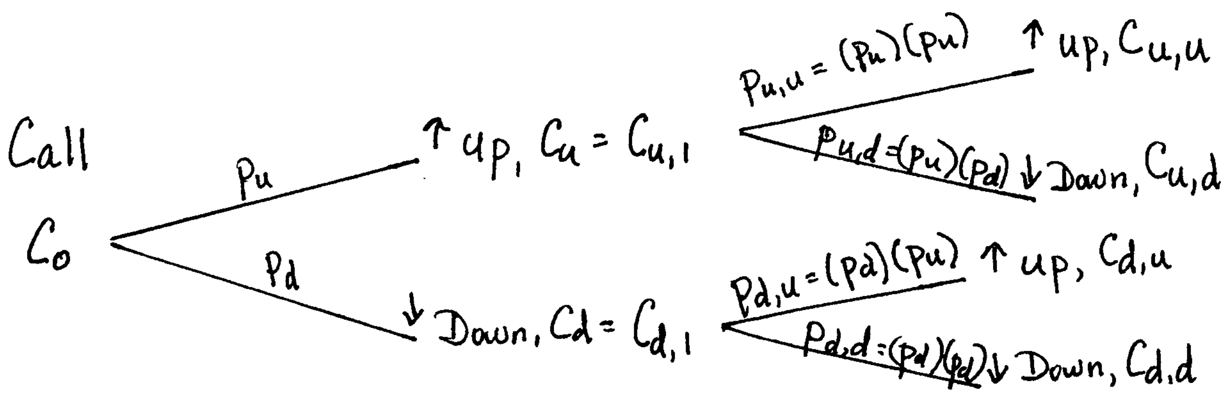
Stock declines 7% from \$42 $\rightarrow$ \$39.06

Two Period Binomial Option Pricing

Extension of one period binomial model to two time periods

Possible Outcomes





2 Period Binomial Model

Call pricing outcomes:

$$C_{u,u} = Max\{0, S_0(1+u)^2 - X\}$$
$$C_{u,d} = Max\{0, S_0(1+u)(1+d) - X\}$$
$$C_{d,u} = Max\{0, S_0(1+d)(1+u) - X\}$$
$$C_{d,d} = Max\{0, S_0(1+d)^2 - X\}$$

Expected Values for the Call Outcomes – using tree Diagram:

looking at the 2nd period

$$C_{u,1} = \frac{p_u \cdot C_{u,u} + p_d \cdot C_{u,d}}{(1+r_f)}$$

$$C_{d,1} = \frac{p_u \cdot C_{d,u} + p_d \cdot C_{d,d}}{(1+r_f)}$$

From 1st period

$$C_0 = \frac{p_u \cdot C_{u,1} + p_d \cdot C_{d,1}}{(1+r_f)}$$

Combining these rel^ships:

$$C_0 = \frac{p_{u,u} \cdot C_{u,u} + 2(p_u)(p_d)\, C_{u,d} + p_{d,d} \cdot C_{d,d}}{(1+r_f)^2}$$

$$C_0 = \frac{(p_u)^2\left[(1+u)^2 S_0 - X\right] + 2(p_u)(p_d)\left[(1+u)(1+d)S_0 - X\right] + \left[(1+d)^2 S_0 - X\right](p_d)^2}{(1+r_f)^2}$$

2 Period Binomial Pricing

Portfolio Hedge

With a 2 period model - the hedge ratios need to be re-adjusted for each time period. This theme of re-adjusting positions will apply to the practical application of portfolio hedges

At the end of the first period, the stock price will be either $S_{u,1}$ or $S_{d,1}$ $\Rightarrow$ new hedge ratio will be $h_{u,1}$ or $h_{d,1}$

$$h_{0,1} = \frac{C_{u,1} - C_{d,1}}{S_u - S_d}$$

$$h_{u,1} = \frac{C_{u,u} - C_{u,d}}{S_{u,u} - S_{u,d}}$$

$$h_{d,1} = \frac{C_{u,d} - C_{d,d}}{S_{u,d} - S_{d,d}}$$

Black Scholes Option Pricing (BSOP)

5 Basic Assumptions Underlying (BSOP)

① Stock price returns are normally distributed, random variables, and the quantity S_{t+1}/S_t is lognormally distributed.

Stock prices follow a normal distribution

② The Stock's variance and the risk-free rate are constant throughout the life of the option.

Over the short life of the option the risk-free rate & stock variance remains invariant.

③ No taxes or transactions costs are paid.

④ The stock pays no dividends.

Two difficulties – ① if dividends are paid, it may be optimal to exercise the option early

② when dividends exist the BSOP model will underestimate the option's price.

⑤ The call must be a European-style option.

The model works only if the option is exercisable at expiration – since early exercise is only optimal when dividends are paid – this assumption is necessary only for dividend paying firms when the option is alive.

The Synthetic Call Option

A Synthetic Call is a portfolio securities that produces the exact same payoff pattern as the call option.

For non dividend paying stocks it is never optimal to exercise early - so the call is exercised at expiration and its value will be equal to 0 or S-X.

- Creation of a Synthetic Call helps to define the actual Value of the Call.

The Synthetic option consists of a long stock position, So and a short position in a $1 par zero coupon bond Bo

At expiration the value of the synthetic call is

$$V_t^* = \text{Max}[0, S_t^* - B_t^*]$$

V_t^* = Value of the Synthetic option
S_t^* = Value of Stock at expiration
B_t^* = maturity value of the zero at t.

Let H_0 = # shares of stock held long when the synthetic position is initiated

N_0 = # \$1 par, zero bonds issued to finance the purchase of stock

The Value of the Synthetic is

$$V_0 = H_0 S_0 - N_0 B_t^* e^{-rt}$$

V_0 has cash flows equivalent to a call option.

With the Black Schole Option Pricing Model

$$V_0 = C_0 = S_0 N(d_1) - X e^{-rt} N(d_2)$$

↑ $N(d_1)$: H_0 = hedge ratio for stock portfolio

↑ $N(d_2)$: N_0 = # Zero Coupon Bonds

$N(d_1)$ & $N(d_2)$ are the probabilities of observing a standard normal, random variable within a given range.

- $N(d_1)$ = prob. of the call expiring in the money
  $N(d_2)$ = prob of paying the Strike price

  d_1 & d_2 Correspond to the z-statistic used with the Normal Distribution $z = \frac{(x - \mu_x)}{\sigma_x}$

$$d_1 = \frac{\ln(S_0/x) + (r_f + \frac{1}{2}\sigma_x^2)(t/365)}{\left[\sqrt{(t/365)}\right]\sigma_x}$$

$$d_2 = d_1 - \left(\sqrt{t/365}\right)\sigma_x$$

- E.G. Given the January 19, 2006 Call option listings for CSCO, assuming the standard deviation of Cisco Stock is 25% use the BSOP model to calculate the theoretical value of the April 17.50 Call.

  Stock price on CSCO = \$ 19.02

  90 day T-Bill Rate = 4.21%

  Dates 1/19/2006 → 4/21/2006

  Using HP 12C # days = 92 days

$$C_0 = 19.02\,N(d_1) - 17.50\,e^{-.0421(92/365)}\,N(d_2)$$

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on

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January 19, 2006

Cisco Systems Inc. (CSCO)

At 4:00PM ET: **19.02**

CALL OPTIONS

Expire at close Fri, Apr 21, 2006

Strike	Symbol	Last	Chg	Bid	Ask	Vol	Open Int
2.50	CYQDZ.X	**15.30**	**0.00**	16.50	16.60	21	907
5.00	CYQDA.X	**14.30**	**0.00**	14.00	14.20	10	157
7.50	CYQDU.X	**10.40**	**0.00**	11.50	11.70	5	673
10.00	CYQDB.X	**9.30**	**0.00**	9.10	9.20	5	662
12.50	CYQDV.X	**6.90**	**0.00**	6.60	6.80	13	1,296
15.00	CYQDC.X	**4.30**	[illegible]	4.20	4.30	100	11,971
17.50	CYQDW.X	**1.95**	[illegible]	1.95	2.05	2,007	37,210
20.00	CYQDD.X	**0.55**	[illegible]	0.55	0.60	2,296	63,499
22.50	CYQDX.X	**0.15**	[illegible]	0.10	0.15	144	18,806
25.00	CYQDE.X	**0.05**	**0.00**	N/A	0.10	50	1,506
27.50	CYQDY.X	**0.05**	**0.00**	N/A	0.05	20	45
30.00	CYQDF.X	**0.05**	**0.00**	N/A	0.05	0	11
32.50	CYQDT.X	**0.05**	**0.00**	N/A	0.05	1	12
35.00	CYQDG.X	**0.05**	**0.00**	N/A	0.05	0	11
37.50	CYQDS.X	**0.05**	**0.00**	N/A	0.05	0	11

ET: 19.02

Thursday, January 19, 2006, 9:56PM ET - U.S. Markets Closed. Dow +0.24% Nasdaq +0.97%

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Enter Symbol(s): e.g. YHOO, ^DJI GO Symbol Lookup | Finance Searc

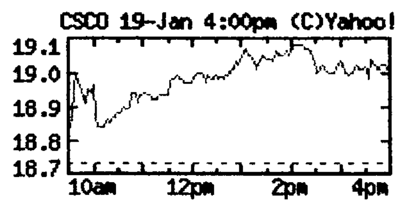
Cisco Systems Inc. (CSCO)

At 4:00PM ET: **19.02**

CISCO SYS INC (NasdaqNM:CSCO) Delayed quote data Edit

After Hours (RT-ECN): **19.02 0.00 (0.00%)**

Last Trade:	**19.02**	Day's Range:	18.82 - 19.12
Trade Time:	4:00PM ET	52wk Range:	16.83 - 20.25
Change:	↑0.29 (1.55%)	Volume:	54,525,205
Prev Close:	18.73	Avg Vol (3m):	58,362,800
Open:	18.84	Market Cap:	116.83B
Bid:	0.01 x 100	P/E (ttm):	22.06
Ask:	9,000.00 x 100	EPS (ttm):	0.86
1y Target Est:	21.82	Div & Yield:	N/A (N/A)



1d 5d 3m 6m 1y 2y 5y max

Annual Report for CSCO

Add CSCO to Portfolio Set Alert Download Data

HEADLINES Change Display [hide $$ edit]

- Cramer's 'Mad Money Lightning Round': AMD's Advance
  at TheStreet.com (Thu 7:36pm)
- Scientific-Atlanta 2Q Profit Declines
  AP (Thu 5:48pm)
- Scientific-Atlanta Misses on Sales
  at TheStreet.com (Thu 4:32pm)
- UPDATE - Scientific-Atlanta posts lower quarterly profit
  at Reuters (Thu 4:27pm)
- Byte and Switch Insider Reports on Block-Level Storage Virtualization
  PR Newswire (Thu 2:53pm)
- Foundry Networks Seen Pressured By Cisco Systems
  at Forbes.com (Thu 1:58pm)
- Citrix Has Plenty Of Momentum For 2006
  at Forbes.com (Thu 1:03pm)
- Apple, eBay outlooks fail to meet expectations
  Reuters (Thu 4:57am)
- Cisco appoints new top marketing executive
  at Reuters (Wed 8:40pm)
- Cisco Announces Leadership Evolution to Support Growth Strategy
  Business Wire (Wed 8:23pm)

More Headlines for CSCO...

KEY STATISTICS

Forward P/E (1 yr):	15.98
P/S (ttm):	4.53
Dividend Date:	N/A
Ex-Dividend Date:	N/A

More Key Statistics...

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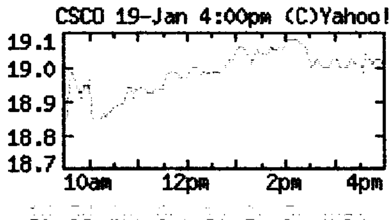

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- Scientific-Atlanta Misses on Sales
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- UPDATE - Scientific-Atlanta posts lower quarterly profit
at Reuters (Thu 4:27pm)
- Byte and Switch Insider Reports on Block-Level Storage Virtualization
PR Newswire (Thu 2:53pm)
- Foundry Networks Seen Pressured By Cisco Systems
at Forbes.com (Thu 1:58pm)
- Citrix Has Plenty Of Momentum For 2006
at Forbes.com (Thu 1:03pm)
-
Reuters (Thu 4:57am)
- Cisco appoints new top marketing executive
at Reuters (Wed 8:40pm)
-
Business Wire (Wed 8:23pm)

More Headlines for CSCO...

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More Key Statistics...

REPORTS

Jan 20 - Market Edge

Trading Report for (CSCO) for Friday + Free Market Timing Report

Jan 19 - Stock Traders Daily

CSCO 3-in-1: Investment Climate Report, Volatility Report, and Correlations

Jan 19 - KRS

More Reports for CSCO

ANALYST

Annual EPS Est (Jul-06) :	1.04
Quarterly EPS Est (Jan-06) :	0.25
Mean Recommendation*:	2.2
PEG Ratio (5 yr expected):	1.16

* (Strong Buy) 1.0 - 5.0 (Strong Sell)

Analyst Opinion - Estimates

BUSINESS SUMMARY

Cisco Systems, Inc. engages in the manufacture and sale of networking and communications products worldwide. The company provides products for transporting data, voice, and video within buildings and across campuses.

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Quotes delayed for CSCO. Get - FREE trial.

Quotes delayed, except where indicated otherwise.
Delay times are 15 mins for NASDAQ, 20 mins for NYSE and Amex. See also delay times for .

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CSCO Call Option

$$d_1 = \frac{\ln\left(\frac{19.03}{17.50}\right) + \left(.0421 + (.5)(.25)^2\right)\left(\frac{92}{365}\right)}{(.25)\left(\sqrt{92/365}\right)}$$

$$d_1 = \frac{.083816 + .049977}{.125513} = 1.065969 \approx 1.07$$

$$d_2 = 1.07 - (.25)\left(\sqrt{92/365}\right) = .944487 \approx .94$$

$N(1.07) = .8577$

$N(.94) = .8261$

$$C_0 = 19.02\,(.8577) - 17.50\,(.989445)(.8261)$$

$$C_0 = 16.31 - 14.30 = 2.01$$

Current Bid / Ask Price on CSCO – Jan 19th 2006

Bid	Ask
1.95	2.05

Sensitivity Analysis

- Variables that Impact BSOP Valuation of the Call premium

① Changes in the underlying stock price
Direct rel^{n}ship between Stock price (S) and the Call premium (C)

$S\uparrow \Rightarrow C\uparrow$
$S\downarrow \Rightarrow C\downarrow$

② Changes in the Strike (exercise) price
Indirect or inverse rel^{n}ship between the exercise price (x) and the Call premium (C)

$X\uparrow \Rightarrow C\downarrow$
$X\downarrow \Rightarrow C\uparrow$

③ Changes in the risk-free rate (exogeneous Variable - influenced by the Fed. Reserve)
Positive rel^{n}ship between risk free rate (r_f) and the Call premium (C)

$r_f\uparrow \Rightarrow C\uparrow$
$r_f\downarrow \Rightarrow C\downarrow$

④ Changes in Time to expiration

Direct relationship between the length of time (t) to expiration and the Call premium (C)

$t\uparrow \Rightarrow C\uparrow$

$t\downarrow \Rightarrow C\downarrow$

⑤ Changes in the Variance of Stock prices

Positive relationship between the stock price Variance (σ_S) and the call premium.

$\sigma_S\uparrow \Rightarrow C\uparrow$

$\sigma_S\downarrow \Rightarrow C\downarrow$

How might BSOP be adapted to the Case of Stocks paying dividends?

Two Approaches

① Changes the market price of the stock to reflect the payment of a dividend - adjust stock price downward.

② Assume a constant dividend yield ⇒ works well with index options because there is no direct underlying security.

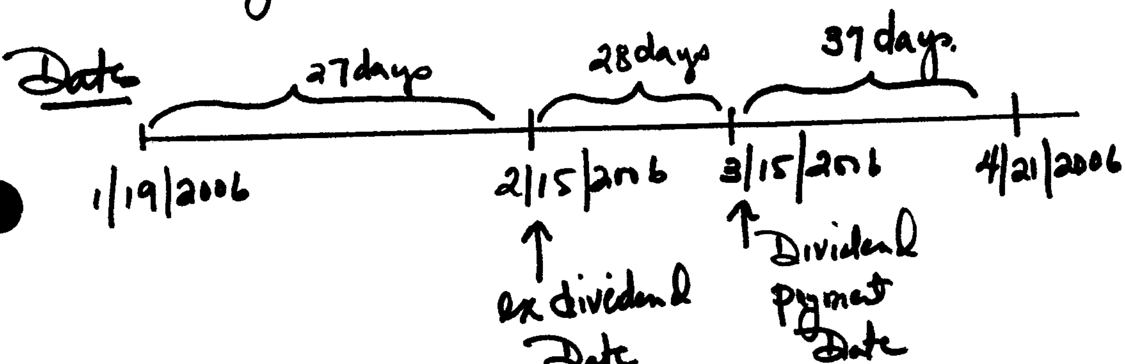
Adjusting the stock price to reflect the payment of dividends: Consider the CSCO option priced on January 19, 2006

Call option – Strike price: \$17.50

$\sigma_{CSCO} = .25$ Dividend = \$.10/shr.

Current price: \$19.02

90 day T-Bill Rate = 4.21%



The exdividend date is date by which the stock must be purchased.

The dividend

① Calculating the present value of the dividend

$$\$.10e^{-.0421(27/365)} = (.10)(.996891)$$

$$= .0997$$

② Subtract the present value of the dividend from the stock price

$$\$19.02 - .0997 = 18.9203$$

The lower adjusted market price of the stock – $18.9203 gets inserted into the BSOP.

A lower call premium results – because the stock price is positively related to the call premium.

$$C_0 = 18.9203\, N(d_1) - 17.50\, e^{-.0421(92/365)} N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{18.9203}{17.50}\right) + (.0421 + (.5)(.25)^2)(92/365)}{(.25)(\sqrt{92/365})}$$

$$d_1 = \frac{.078035 + .018488}{.125513} = .769028$$

$$d_2 = .769028 - \sqrt{(92/365)}\,(.25)$$
$$= .769028 - .125513 = .643515$$

$N(.77) = .7794 \approx .78$

$N(.64) = .7389 \approx .74$

$$C_0 = 18.9203(.78) - 17.50(.989445)(.74)$$
$$C_0 = 14.76 - 12.81 = \$1.95$$

Call premium went from $2.01 to $1.95